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# Exploring the flavour structure of the MSSM with rare $K$ decays

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Abstract: We present an extensive analysis of rare $K$ decays, in particular of the two neutrino modes $K^+ \to \pi^+ \nu\overline{\nu}$ and $K_L \to \pi^0 \nu\overline{\nu}$, in the Minimal Supersymmetric extension of the Standard Model. We analyse the expectations for the branching ratios of these modes, both within the restrictive framework of the minimal flavour violation hypothesis and within a more general framework with new sources of flavour-symmetry breaking. In both scenarios, the information that can be extracted from precise measurements of the two neutrino modes turn out to be very useful in restricting the parameter space of the model, even after taking into account the possible information on the mass spectrum derived from high-energy colliders, and the constraints from $B$-physics experiments. In the presence of new sources of flavour-symmetry breaking, additional significant constraints on the model can be derived also from the two $K_L \to \pi^0 \ell^+ \ell^-$ modes.

Keywords: Kaon Physics, Supersymmetric Standard Model, Rare Decays, Beyond Standard Model.

 

## Contents





## 1. Introduction

As widely discussed in the literature, rare decays dominated by one-loop electroweak dynamics offer a very powerful tool to investigate the flavour structure of physics beyond the Standard Model (SM) [1]. Among them, the processes $K^+ \to \pi^+ \nu\overline{\nu}$ and $K_L \to \pi^0 \nu\overline{\nu}$ are certainly a privileged observatory because of the high-level of accuracy achieved in their theoretical description [2 − 7]: future precise measurements of these modes will have a non-trivial impact on physics well above the electroweak scale [8].

In the last few years precise measurements of flavour-changing neutral-current (FCNC) processes in the $B$ sector have severely restricted the parameter space of new-physics models, especially in the flavour sector. Moreover, a direct exploration of the physics in the TeV range is expected soon with the start of the LHC program. Within this context, it is worth to understand if, and at which level, the indirect information which could be extracted from rare $K$ decays is still useful. The main purpose of the present paper is an attempt to answer this question, within the specific framework of the Minimal Supersymmetric extension of the SM (MSSM).

Several analyses of rare $K$ decays within the MSSM have already been presented in the literature, both in the general framework of arbitrary new sources of flavour mixing [9 − 14], and also in well-motivated scenarios with more restrictive hypotheses [15]. The main purpose of all these works has been the identification of the maximal deviations from the SM of the two $K \to \pi\nu\overline{\nu}$ rates. Our analysis has a different goal: understanding how precise measurements of these observables can be used to discriminate among different versions of the MSSM. We will analyse in particular two general frameworks:

I. The most general version of the MSSM compatible with the Minimal Flavour Violation (MFV) hypothesis, as defined in ref. [16].

II. The MSSM with generic new sources of flavour-symmetry breaking, in particular with sizable non-MFV trilinear soft-breaking terms in the up sector ($\mathbf{A}_U$), with $R$-parity conservation and moderate values of $\tan\beta$ ($\tan\beta \lesssim 30$).

As we will show, in both these frameworks precise measurements of the two $\mathcal{B}(K \to \pi\nu\overline{\nu})$ are very useful to determine the (flavour) structure of the model. This statement remains true even taking into account possible future constraints on the MSSM mass spectrum obtained at the LHC, and the refinement of the flavour constraints expected from $B$ factories. Within the scenario II, we will show in particular that present constraints still allow a large freedom concerning the flavour structure of the $\mathbf{A}_U$ terms. In the presence of sizable deviations from the MFV hypothesis in this sector, a key role is played also by the two $K_L \to \pi^0\ell^+\ell^-$ modes.

The paper is organized as follows: in Section 2 we recall some basic formulae for the evaluation of rare $K$ decay branching ratios. In section 3 we: i) introduce the MFV scenario; ii) analyse the expectations of the two $\mathcal{B}(K \to \pi\nu\overline{\nu})$ in this framework; iii) discuss the consequences of these findings and compare them with the previous literature. Similarly, in section 4 we introduce and analyse the consequences for rare $K$ decays of the scenario II. The main results are summarized in the Conclusions.

## 2. Basic formulae for rare $K$ decays

Within the class of models considered here, the supersymmetric contributions to $K \to \pi\nu\bar{\nu}$ decays can be described to a good accuracy in terms of a single complex function[1]

$$W = \frac{1}{3}\sum_{l=e,\mu,\tau} W_l\,, \tag{2.1}$$

where $W_l$ are the Wilson coefficients of the following effective Hamiltonian:

$$\mathcal{H}_{\rm eff}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}}\frac{\lambda^5\alpha_{\rm em}}{2\pi\sin^2\theta_W}\sum_{l=e,\mu,\tau} W_l\ \bar{s}\gamma^\mu(1-\gamma_5)d\ \bar{\nu}_l\gamma_\mu(1-\gamma_5)\nu_l\ +\ \text{h.c.} \tag{2.2}$$

with $\lambda = |V_{\rm us}| = 0.225 \pm 0.001$. In terms of this function, the two $K \to \pi\nu\bar{\nu}$ branching ratios can be written as

$$\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = \kappa_+\,|W_{\rm SM} + W_{\rm SUSY}|^2 \tag{2.3}$$

$$\mathcal{B}\left(K_L \to \pi^0\nu\bar{\nu}\right) = \kappa_L\,[\mathrm{Im}(W_{\rm SM} + W_{\rm SUSY})]^2 \tag{2.4}$$

[1] There are two notable corners of the MSSM parameter space (which will not be analysed in this work) where this approximation is not valid: the large-$\tan\beta$ scenario with non-MFV right-right mixing terms, discussed recently in [14], and models with large violations of lepton-flavour universality, discussed in [17].

where[2] $\kappa_+ = (5.26 \pm 0.06) \times 10^{-11}$ and $\kappa_L = (2.29 \pm 0.03) \times 10^{-10}$. Defining further $\lambda_q = V_{qs}^* V_{qd}$ (where $V_{ij}$ denotes the generic element of the Cabibbo-Kobayashi-Maskawa matrix), the SM contribution to the $W$ function reads

$$W_{\rm SM} = \frac{{\rm Re}\lambda_c}{\lambda} P_{u,c} + \frac{{\rm Re}\lambda_t}{\lambda^5} X_t + i \frac{{\rm Im}\lambda_t}{\lambda^5} X_t \,, \tag{2.5}$$

with $X_t = 1.464 \pm 0.041$ [3] and $P_{u,c} = 0.41 \pm 0.04$ [2, 4]. For the numerical values of the other input parameters we refer to refs. [18, 19].

As discussed in [9, 10, 12, 11, 13], among the additional non-standard contributions to the $W$ function appearing in the MSSM ($W_{\rm SUSY}$), only those associated with chargino up-squark loops and charged-Higgs top-quark loops can compete in size with $W_{\rm SM}$. A complete listing of these contributions can be found in ref. [13].

The effective Hamiltonian necessary to describe the two $K_L \to \pi^0 \ell^+ \ell^-$ modes can in general be written as

$$\mathcal{H}_{\rm eff}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} \frac{\lambda^5 \alpha_{\rm em}}{2\pi} \Big[ \sum_i w_i^e(\mu) Q_i^e(\mu) + w_i^\mu(\mu) Q_i^\mu(\mu) \Big] + {\rm h.c.} \,, \tag{2.6}$$

where the list of potentially relevant operators includes four-quark operators, photon- and gluon-dipole operators, and

$$Q_{7V}^\ell = \bar{s}\gamma^\mu(1-\gamma_5) d\, \bar{\ell}\gamma_\mu \ell \,, \qquad Q_{7A}^\ell = \bar{s}\gamma^\mu(1-\gamma_5) d\, \bar{\ell}\gamma_\mu\gamma_5 \ell \,. \tag{2.7}$$

Both within the SM and in the class of supersymmetric models we are considering, the direct-CP-violating transition $K_2 \to \pi^0 \ell^+ \ell^-$ turns out to be dominated by the contributions of $Q_{7V,A}^\ell$. In this limit, the corresponding $K_L$ branching ratios can be written as [20]

$$\mathcal{B}(K_L \to \pi^0 \ell^+ \ell^-) = \left( C_{\rm mix}^\ell + C_{\rm int}^\ell + C_{\rm dir}^\ell + C_{\rm CPC}^\ell \right) \times 10^{-12}, \tag{2.8}$$

with the following set of coefficients:

$$\begin{aligned}
&C_{\rm int}^e = -(7.73 \pm 0.20) \times |a_S| \times {\rm Im}\; w_{7V}^e \,, && C_{\rm mix}^e = (14.9 \pm 0.5) \times |a_S|^2 \,, \\
&C_{\rm int}^\mu = -(1.83 \pm 0.04) \times |a_S| \times {\rm Im}\; w_{7V}^\mu \,, && C_{\rm mix}^\mu = (3.54 \pm 0.2) \times |a_S|^2 \,, \\
&C_{\rm dir}^e = (2.12 \pm 0.10) \times \left[ ({\rm Im}\; w_{7V}^e)^2 + ({\rm Im}\; w_{7A}^e)^2 \right] \,, && C_{\rm CPC}^e \approx 0 \,, \\
&C_{\rm dir}^\mu = (1.17 \pm 0.05) \times \left[ 0.43 \times ({\rm Im}\; w_{7V}^\mu)^2 + ({\rm Im}\; w_{7A}^\mu)^2 \right] \,, && C_{\rm CPC}^\mu = (5.2 \pm 1.6) \,,
\end{aligned} \tag{2.9}$$

Here $|a_S| = 1.2 \pm 0.2$ denotes the non-perturbative low-energy constant extracted from the experimental value of $\mathcal{B}(K_S \to \pi^0 \ell^+ \ell^-)$ [21], and the Wilson coefficients are renormalized at the scale $\mu_{\rm IR} \approx 1\,{\rm GeV}$. Following the analyses of [20, 22], we have assumed a positive interference between the long-distance amplitude and the SM short-distance contribution. Using the notation of [24], the latter can be expressed as

$$\left( {\rm Im} w_{7V}^\ell \right)_{\rm SM} = -y_{7V} \times \frac{2\pi {\rm Im}\lambda_t}{\lambda^5 \alpha_{\rm em}} = -\,(0.73 \pm 0.04) \times \frac{2\pi {\rm Im}\lambda_t}{\lambda^5} \,, \tag{2.10}$$

$$\left( {\rm Im} w_{7A}^\ell \right)_{\rm SM} = -y_{7A} \times \frac{2\pi {\rm Im}\lambda_t}{\lambda^5 \alpha_{\rm em}} = (0.68 \pm 0.03) \times \frac{2\pi {\rm Im}\lambda_t}{\lambda^5} \,. \tag{2.11}$$

[2] The numerical values of the $\kappa_i$ –which encode isospin-breaking and SU(3) violations in the $K \to \pi$ matrix elements [7]– have been updated with respect to the previous literature taking into account the latest results on the $K \to \pi$ form factors from ref. [18]. The same comment applies to the $C_i^\ell$ in eq. (2.9).

The complete analytic expressions of the chargino up-squark loop contributions to $w^{\ell}_{7V,A}$ can be found in [23].

## 3. The MFV framework

### 3.1 Definition of the model

The MSSM with $R$ parity conservation and generic supersymmetry soft-breaking terms has a huge number of free parameters. One of the virtues of $\mathcal{B}(K \to \pi\nu\bar{\nu})$ is that these observables are sensitive only to a limited subset of such parameters: those appearing in the chargino and up-squark mass matrices, and those which determine the charged-Higgs mass. To define the structure of the MSSM we are considering, it is therefore sufficient to specify the value of these mass terms.

The chargino mass matrix in the basis of electroweak eigenstates (wino and higgsino) is

$$M_\chi = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}, \tag{3.1}$$

where the first index of both rows and columns refers to the wino state. Here $\mu$ denotes the supersymmetric Higgs bilinear coupling, $M_2$ the soft supersymmetry-breaking wino mass and $\tan\beta = v_u/v_d$ the ratio of the two Higgs vacuum expectation values. Note that $M_2$ can always be chosen real, without loss of generality.

Two constraints on the free parameters of $M_\chi$ could be obtained by measuring chargino masses, while independent information could be extracted from the cross sections of various electroweak processes. It is therefore reasonable to assume that the complete structure of $M_\chi$ will be determined, to a good extent, by high-energy experiments.[3] The situation is very different in the squark sector, where the large number of free parameters does not allow a model-independent extraction in terms of high-energy data only.

The soft-breaking terms appearing in the squark sector are the $3 \times 3$ matrices $\mathbf{M}^2_Q$, $\mathbf{M}^2_U$, $\mathbf{M}^2_D$ (bilinear terms), and $\mathbf{A}_U$, $\mathbf{A}_D$ (trilinear terms). Performing in the squark sector the same unitary rotations which allow to diagonalize quark mass matrices,

$$\tilde{\mathbf{q}}_{L,R} \;\to\; \mathbf{V}^q_{L,R}\tilde{\mathbf{q}}_{L,R} \qquad V_{\rm CKM} = \mathbf{V}^u_L \mathbf{V}^{d\dagger}_L \tag{3.2}$$

(i.e. adopting the so-called super-CKM basis), the $6 \times 6$ squark mass matrices assume the form

$$\mathbf{M}^2_{\tilde{u}} \;\to\; \hat{\mathbf{M}}^2_{\tilde{u}} = \begin{pmatrix} \hat{\mathbf{M}}^2_{\tilde{u}LL} & \hat{\mathbf{M}}^{2\dagger}_{\tilde{u}LR} \\ \hat{\mathbf{M}}^2_{\tilde{u}LR} & \hat{\mathbf{M}}^2_{\tilde{u}RR} \end{pmatrix}, \qquad \mathbf{M}^2_{\tilde{d}} \;\to\; \hat{\mathbf{M}}^2_{\tilde{d}} = \begin{pmatrix} \hat{\mathbf{M}}^2_{\tilde{d}LL} & \hat{\mathbf{M}}^{2\dagger}_{\tilde{d}LR} \\ \hat{\mathbf{M}}^2_{\tilde{d}LR} & \hat{\mathbf{M}}^2_{\tilde{d}RR} \end{pmatrix}, \tag{3.3}$$

[3] An additional significant information is obtained by flavour-conserving low-energy experiments, in particular by the electric-dipole-moments (e.d.m.) of quarks and leptons, which already provide stringent constraints on the possible CP violating phase of $\mu$ [25].



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where

$$\hat{\mathbf{M}}^2_{\tilde{u}LL} = \mathbf{V}^u_L \mathbf{M}^2_Q \mathbf{V}^{u\dagger}_L + \mathbf{m}^2_u + \mathbf{1}\frac{1}{6}(4M^2_W - M^2_Z)\cos 2\beta\,, \tag{3.4}$$

$$\hat{\mathbf{M}}^2_{\tilde{u}RR} = \mathbf{V}^u_R \mathbf{M}^2_U \mathbf{V}^{u\dagger}_R + \mathbf{m}^2_u + \mathbf{1}\frac{2}{3}M^2_Z \cos 2\beta \sin^2\theta_W\,, \tag{3.5}$$

$$\hat{\mathbf{M}}^2_{\tilde{u}LR} = v_u \mathbf{V}^u_R \mathbf{A}_U \mathbf{V}^{u\dagger}_L - \cot\beta\mu^* \mathbf{m}_u\,, \tag{3.6}$$

with $\mathbf{m}_u = \mathrm{diag}(m_u, m_c, m_t)$, and similarly for the down sector. For later convenience, we also define the couplings

$$\left(\delta^U_{\mathrm{AB}}\right)_{ij} = \frac{(\mathbf{M}^2_{\tilde{u}AB})_{ij}}{|(\mathbf{M}^2_{\tilde{u}AA})_{ii}|^{1/2}|(\mathbf{M}^2_{\tilde{u}BB})_{jj}|^{1/2}} \qquad (A, B = L, R) \tag{3.7}$$

which parametrize the amount of flavour-symmetry breaking in the up-squark sector (in a generic squark basis).

The flavour structure both of the SM and the MSSM is characterized by a global $\mathrm{SU}(3)^5$ flavour symmetry, broken only by mass terms and Yukawa interactions [16]. Within the SM, the Yukawa interaction is the only source of $\mathrm{SU}(3)^5$ breaking. Within the MSSM, there are in general several new flavour-symmetry breaking sources, encoded in the soft-breaking terms. In the following we shall concentrate on the MFV scenario, which can be considered as the most restrictive assumption about the structure of these additional flavour-symmetry breaking terms.

According to the MFV hypothesis, the SM Yukawa matrices are the only source of breaking of the $\mathrm{SU}(3)^5$ flavour symmetry also beyond the SM [16]. Neglecting suppressed terms (proportional to high powers of off-diagonal CKM terms and/or light quark masses), this symmetry principle implies the following structure for the soft-breaking terms of the quark sector (in a generic basis for the electroweak eigenstates) [16]:

$$\begin{aligned}
\mathbf{M}^2_Q &= \tilde{m}^2\left(\tilde{a}_1\mathbf{1} + \tilde{b}_1\mathbf{Y}^\dagger_u\mathbf{Y}_u + \tilde{b}_2\mathbf{Y}^\dagger_d\mathbf{Y}_d + \tilde{b}_3\left(\mathbf{Y}^\dagger_d\mathbf{Y}_d\mathbf{Y}^\dagger_u\mathbf{Y}_u + \mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{Y}_d\right)\right)\\
\mathbf{M}^2_U &= \tilde{m}^2\left(\tilde{a}_2\mathbf{1} + \tilde{b}_4\mathbf{Y}_u\mathbf{Y}^\dagger_u\right)\,, \qquad \mathbf{A}_U = A\mathbf{Y}_u\left(\tilde{a}_4\mathbf{1} + \tilde{b}_6\mathbf{Y}^\dagger_d\mathbf{Y}_d\right)\\
\mathbf{M}^2_D &= \tilde{m}^2\left(\tilde{a}_3\mathbf{1} + \tilde{b}_5\mathbf{Y}_d\mathbf{Y}^\dagger_d\right)\,, \qquad \mathbf{A}_D = A\mathbf{Y}_d\left(\tilde{a}_5\mathbf{1} + \tilde{b}_7\mathbf{Y}^\dagger_u\mathbf{Y}_u\right)\,,
\end{aligned} \tag{3.8}$$

where $\tilde{a}_i$ and $\tilde{b}_i$ are free $\mathcal{O}(1)$ parameters. Redefining these parameters absorbing the dimensional factors $\tilde{m}$ and $A$, this implies for the up squark mass matrix ($V \equiv V_{\mathrm{CKM}}$)

$$\begin{aligned}
\hat{\mathbf{M}}^2_{\tilde{u}LL} &= a^2_1\mathbf{1} + \frac{b^2_1}{v^2_u}\mathbf{m}^2_u + \frac{b^2_2}{v^2_d}V\mathbf{m}^2_dV^\dagger + \frac{b^2_3}{v^2_uv^2_d}\left(V\mathbf{m}^2_dV^\dagger\mathbf{m}^2_u + \mathbf{m}^2_uV\mathbf{m}^2_dV^\dagger\right)\\
&\quad + \mathbf{m}^2_u + \mathbf{1}\frac{1}{6}(4M^2_W - M^2_Z)\cos 2\beta\,,\\
\hat{\mathbf{M}}^2_{\tilde{u}RR} &= a^2_2\mathbf{1} + \frac{b^2_4}{v^2_u}\mathbf{m}^2_u + \mathbf{m}^2_u + \mathbf{1}\frac{2}{3}M^2_Z\cos 2\beta\sin^2\theta_W\,,\\
\hat{\mathbf{M}}^2_{\tilde{u}LR} &= a_4\mathbf{m}_u + \frac{b_6}{v^2_d}\mathbf{m}_uV\mathbf{m}^2_dV^\dagger - \cot\beta\mu^*\mathbf{m}_u\,,
\end{aligned} \tag{3.9}$$

in the super-CKM basis, and similarly for the down squark mass matrix. Note that in principle $a_{4,5}$ and $b_{6,7}$ can be complex.

| Ranges (GeV) | | Exp. bounds (GeV) | |
|---|---|---|---|
| $a_{1-3}$ : | [0, 1000] | $M_{\chi^{\pm}_{1,2}} > 94$ | $M_{\tilde{c},\tilde{u},\tilde{s},\tilde{d}} > 250$ |
| $\|a_4\|$ : | [0, 3000] | $M_{\chi^0_1}$ $(M_{\chi^0_2}) > 46$ (63) | $M_{\tilde{t}} > 96$ |
| $\|\mu\|$ : | [0, 500] | $M_{\chi^0_3}$ $(M_{\chi^0_4}) > 100$ (116) | $M_{\tilde{b}} > 89$ |
| $M_2$ : | [0, 3000] | | |

**Table 1:** Ranges adopted in the scan for the free parameters of the MFV framework relevant for $K \to \pi\nu\overline{\nu}$. Experimental lower bounds on squark and gaugino masses [29].

### 3.2 Scanning of the parameter space

Employing the parametrizations in eqs. (3.9) and (3.1) we have performed a systematic scan of the free parameters of the model and analysed the consequences for the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$. Within the MFV framework, the deviations with respect to the SM are experimentally undetectable in a good fraction of the parameter space. For this reason, we concentrated in particular to identify under which conditions (within this restricted scenario) it is possible to generate sizable (detectable) enhancements with respect to the SM in the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$. The free parameters have been varied in a wide range, checking the consistency with tree-level vacuum stability bounds [26], direct experimental constraints on squark and gaugino masses (see table 1) and existing constraints from precision measurements (both in the electroweak and in the flavour sector).

The scan has been performed using a combination of numerical methods to optimize the search of maximal effects and to deal most effectively with invalid regions in parameter space. Besides the numerical integration routine VEGAS [27] –whose use for adaptive scanning was proposed in ref. [28]– simple random scans together with various redefinitions of the parameter space were necessary to probe the interesting regions. Indeed, as we will see, the largest effects always occur on the boundary of the allowed parameter space, a situation in which VEGAS can be highly inefficient or even misleading. For these reasons, simple but extensive scans (several millions of points) were first performed to identify the interesting regions; in a second step, VEGAS was used to scan inside these regions (still with high damping and a few iterations).

The chosen ranges of the relevant parameters are reported in table 1. The parameter $a_5$ and all the $b_i$ have a negligible effect on the considered branching ratios and can be set to zero without loss of generality. The MFV implementation of vacuum stability bounds then assumes the simple form $|a_4|^2 \leq 3(a_1^2 + a_2^2)$. Also $a_3$ has a very small impact, but has to be taken sufficiently large to generate down squark masses large enough to pass the experimental bounds. A similar comment applies to the gaugino mass parameter $M_1$ in connection with the experimental bounds on neutralino masses. For definiteness, we have taken $M_1 = 500\,\text{GeV}$. As far as $\tan\beta$ is concerned, we have fixed it to the reference value $\tan\beta = 2$: as we will discuss in the following, larger/smaller values of $\tan\beta$ lead to smaller/larger effects in $\mathcal{B}(K \to \pi\nu\bar{\nu})$.[4] The impact of the phases of $\mu$ and $a_4$ has

[4] Low values of $\tan\beta$ are strongly constrained by the experimental constraints on the Higgs sector, and

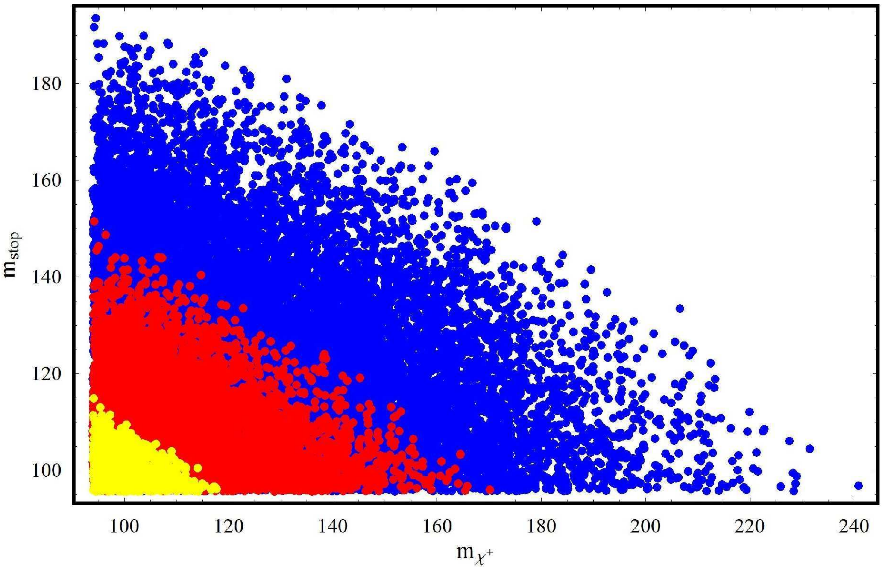


**Figure 1:** Regions in the $m_{\tilde{t}} — m_{\tilde{\chi}}$ plane (lightest stop and chargino masses) allowing enhancements of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ of more than 11% (yellow/light gray), 8.5% (red/medium gray) and 6% (blue/dark gray) in the MFV scenario, for $\tan\beta = 2$ and $M_{H^+} > 1\,\text{TeV}$ [the corresponding enhancements for $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ are 15%, 12.5% and 10%, respectively, see eq. (3.10)].

also been explicitly studied, and found to be quite small (we will come back to this point later), so we considered only real parameters in the final scan. In any case, the different sign combinations of $\mu$ and $a_4$ allow to keep track of these phase effects. Finally, in order to disentangle the various contributions, we have set $M_{H^+} > 1\,\text{TeV}$, such that the charged-Higgs top-quark loops decouple. As chargino box effects can be safely neglected for a lightest slepton mass above 300 GeV, in practice we are left only with the chargino up-squark $Z$ penguin.

The main results of the scan are illustrated in figures 1 and 2. As shown in figure 1, the maximal enhancements of the branching ratios are closely correlated to the minimal values for the lightest stop and chargino masses.[5] This correlation is very useful to determine the flavour structure of the model. For instance, if $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ was found to be more than 10% above the SM expectation and the lightest stop and chargino masses were both found to be above 130 GeV, with a charged Higgs mass above 1 TeV, then one could exclude the MFV scenario. As expected, the MFV hypothesis predicts also a strict correlation between $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$. We find in particular

$$R(K^+ \to \pi^+\nu\bar{\nu}) = (0.965 \pm 0.008) \times R(K_L \to \pi^0\nu\bar{\nu})\,, \tag{3.10}$$

$$R(K \to f) = \mathcal{B}(K \to f)/\mathcal{B}(K \to f)_{\rm SM}\,, \tag{3.11}$$

in particular by the lower bounds on the lightest neutral Higgs [30]. We have explicitly checked that for $\tan\beta = 2$ (and even slightly below) we are compatible with these experimental bounds. This happens mainly because we allow sizable trilinear soft-breaking terms in the up sector.

[5] The variation of density in these plots is not strictly correlated to the density of the underlying parameter space: by construction, we scan with more points the regions with larger effects.

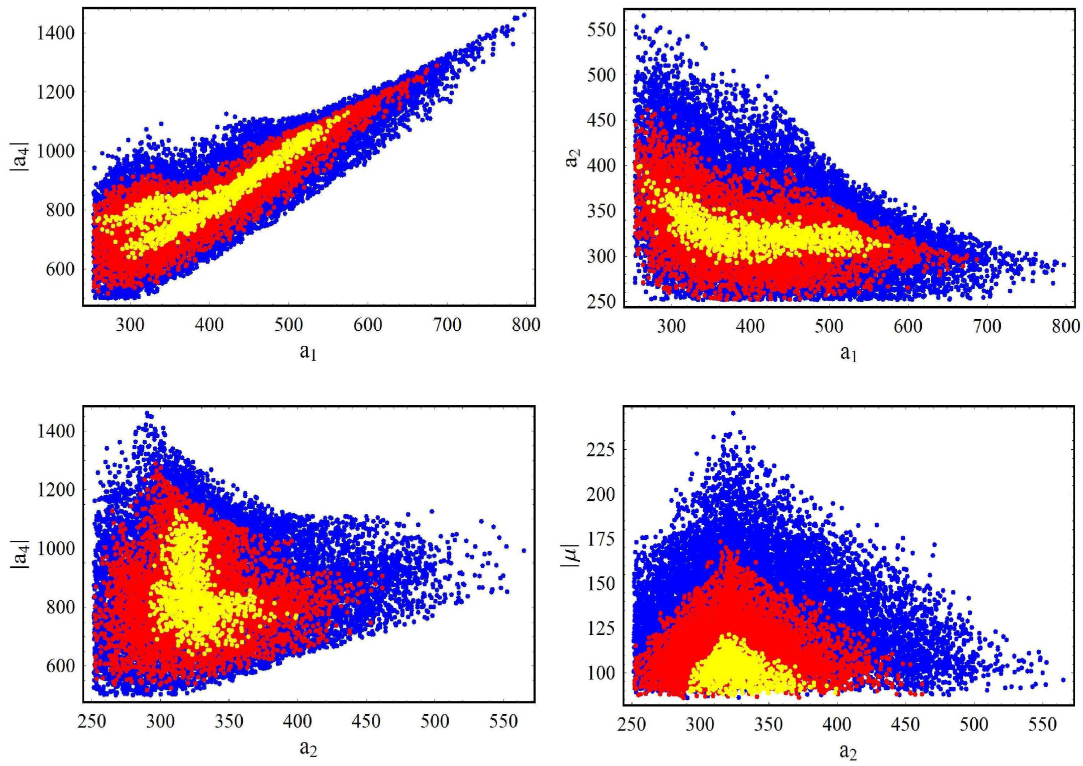


**Figure 2:** Correlations of the most significant MSSM-MFV parameters corresponding to different enhancements of $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ (identified by the color/gray-scale as in figure 1).

in the region of maximal enhancements (i.e. 10% to 16% for the neutral mode). In principle, the relation (3.10) would allow the best test of the MFV hypothesis. However, the experimental challenges of the $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ mode make the correlation between $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$, $m_{\tilde{t}}$ and $m_{\tilde{\chi}}$ outlined in figure 1 a more useful test for the near future.

In figure 2, we present a more detailed analysis of the parameter-space region with enhanced $\mathcal{B}(K \to \pi\nu\bar{\nu})$, showing the two-dimensional projections on the most significant planes. In this case is even more evident the key role of a precision measurement of $\mathcal{B}(K \to \pi\nu\bar{\nu})$ in selecting a well-defined region of the model, or in constraining its structure. As can be noted, an important role is played by the parameter $a_4$: sizable enhancements of $\mathcal{B}(K \to \pi\nu\bar{\nu})$ can occur only for large enough values of this parameter. The reason of this effect can be traced back to the enhancement mechanism discussed in ref. [11]. Indeed, even within the MFV scenario one generates non-vanishing left-right flavour-mixing terms in the squark basis of refs. [10, 11]. In particular, the double mass-insertion combination which controls possible enhancements in $\mathcal{B}(K \to \pi\nu\bar{\nu})$ [11, 13] assumes the form[6]

$$\left(\bar{\delta}^U_{\rm RL}\right)^*_{32} \left(\bar{\delta}^U_{\rm RL}\right)_{31} \propto m_t^2 V^*_{ts} V_{td} \, |a_4^* - \mu \cot\beta|^2 \tag{3.12}$$

and thus grows with $a_4$. The expression (3.12) also shows that: i) a possible phase of $a_4$ does not induce extra CP violation ($\mu$ is approximately real, as implied by the stringent e.d.m. bounds); ii) maximal effects are obtained for $\mu$ and $a_4$ of opposite signs iii) small values of $\tan\beta$ enhance the magnitude of $(\delta^U_{\rm RL})^*_{32}(\delta^U_{\rm RL})_{31}$ still further. We finally note that the contribution of $\mu$ is always subleading with respect to the one of $a_4$ (and thus the sensitivity to $\tan\beta$ and the relative phase between $a_4$ and $\mu$ is quite mild) since large values of $\mu$ lead to a strong suppression of the loop function (because of heavy higgsino masses).

The negligible impact of the $b_i$ parameters, in particular of $b_6$, can also be easily understood in terms of the double mass insertion in eq. (3.12). Indeed, keeping the explicit dependence on $b_6$, leads to the following replacement in eq. (3.12):

$$|a_4^* - \mu \cot\beta|^2 \quad\longrightarrow\quad \left(a_4^* - \mu \cot\beta + b_6^* \frac{m_s^2}{v_d^2}\right) \left(a_4 - \mu^* \cot\beta + b_6 \frac{m_d^2}{v_d^2}\right) \ . \tag{3.13}$$

As can be seen, the $b_6$ contribution is clearly suppressed by the smallness of $m_{d,s}$ (even for large values of $\tan\beta$). Since the internal stop is essentially right-handed, the only other relevant $b_i$ parameter is $b_4$. However, for our purposes, the effect of $b_4$ is equivalent to a redefinition of $a_2$ and thus can be safely neglected.

As far as the constraints from other observables are concerned, the MFV hypothesis automatically implies small non-standard effects in FCNC observables such as $\epsilon_K$ and $\mathcal{B}(B \to X_s\gamma)$. Moreover, these observables are sensitive to different combinations of free parameters with respect to the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$. In particular, the chargino contributions to $Z$ penguins, $\Delta F = 2$ boxes and photon-dipole amplitudes are largely uncorrelated. As a result, the requirement of consistency with existing FCNC data (including $\epsilon_K$ and $\mathcal{B}(B \to X_s\gamma)$, for which we require relative non-standard contributions not exceeding 10% and 15%, respectively) does not have a perceptible impact on figures 1 and 2. As pointed out in ref. [15], an important constraint is obtained from flavour-conserving electroweak observables and, in particular, from $\Delta\rho$ (this is not surprising since the leading chargino contribution to $\Delta\rho$ is very similar – but for the flavour structure- to the $Z$-penguin amplitude contributing to $K \to \pi\nu\bar{\nu}$). The correlation between $\Delta\rho$ and $\mathcal{B}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ is illustrated in figure 3. The effect for $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ is almost identical, provided one scales the enhancement according to eq. (3.10).

### 3.3 Comparison with previous literature

We conclude the MFV analysis with a comparison of our results with those obtained in a similar framework in refs. [15] and [31]. First of all, we recall that for simplicity the

[6] We denote by $(\bar{\delta}^U_{\rm LR})_{ij}$ the flavour-mixing couplings of eq. (3.7) in the squark basis of refs. [10, 11]. Note that the mass insertion approximation is invoked here only to clarify the discussion: squark mass matrices have been diagonalized exactly throughout this work.

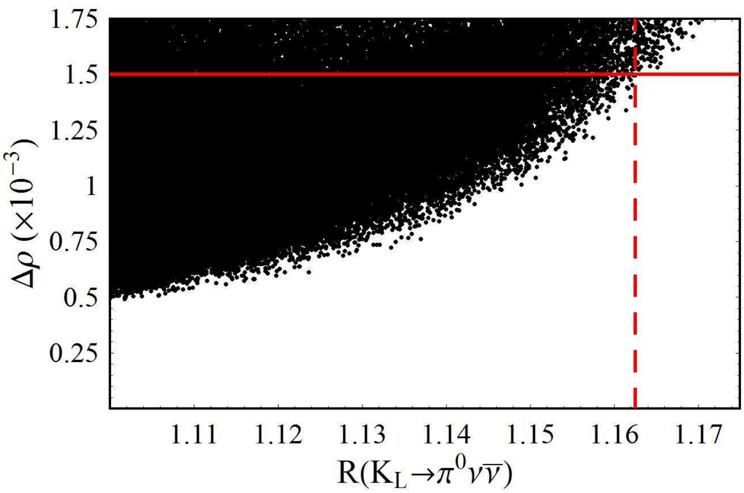


**Figure 3:** Correlation between $\Delta\rho$ and $\mathcal{B}(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ in the MFV framework (with $\tan\beta = 2$ and $M_{H^+} > 1\,\text{TeV}$).

numerical results in figures 1 and 2 have been obtained in the limit where the charged-Higgs contribution can be neglected. This approximation can easily be removed: charged-Higgs top-quark loops always induce a *constructive* contribution to the $W$ function, which depends only on $\tan\beta$ and $M_{H^+}$ (see e.g. ref. [11]). For $\tan\beta = 2$, the maximal effect amounts to an additional $\approx 5\%$ enhancement of the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$ (whose limiting factor is the $M_{H^+} \gtrsim 300\,\text{GeV}$ bound derived from $B \to X_s\gamma$). As a result, we conclude that in the MFV framework

$$R(K^+ \to \pi^+ \nu\bar{\nu}) \lesssim 1.20\,, \qquad R(K_{\rm L} \to \pi^0 \nu\bar{\nu}) \lesssim 1.25\ . \tag{3.14}$$

This conclusion is in contradiction with respect to the claim of ref. [15] that no sizable enhancement of the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$ is possible in the MFV framework. The origin of this difference is twofold:

- We have considered a more general definition of the MFV framework (the most general definition compatible with the renormalization group). Having a more general structure for the soft-breaking terms, within our scheme the effects in $\mathcal{B}(K \to \pi\nu\bar{\nu})$ are less severely constrained by the existing constraints on other observables. In particular, it is worth stressing that: i) the difference between our implementation of the MFV hypothesis and the one adopted in ref. [15] holds also in the limit where the $b_i$ are neglected; ii) within our scheme chargino contributions to $Z$ penguins, $\Delta F = 2$ boxes and photon-dipole amplitudes turn out to be largely uncorrelated.

- We have adopted a different strategy concerning the fit of the CKM matrix. In our analysis we have implicitly assumed that within the MFV framework the determination of the CKM matrix is not affected by the presence of new physics. Thanks to the recent precise results of $B$ factories, we now know that this assumption is an excellent approximation (see e.g. [32]). At the time of ref. [15], the available experimental information was less precise and the authors decided to perform a non-standard CKM fit using observables sensitive to SUSY corrections. This fact introduced spurious correlations between the genuine SUSY effects and indirect effects associated with the CKM determination. In particular, the predictions of the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$ turned out to be suppressed because of smaller effective values of $\lambda_t$, and not because of suppressed loop functions.

In conclusion, we find that in the MSSM realization of the MFV hypothesis the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$ can saturate the model-independent bounds of ref. [31]. This implies that a measurement of one of the two $\mathcal{B}(K \to \pi\nu\bar{\nu})$ consistent with eq. (3.14) does not allow to distinguish a generic MFV model from the MSSM. On the other hand, as illustrated by figure 1, stringent tests of the model can be performed combining $\mathcal{B}(K \to \pi\nu\bar{\nu})$ and sparticle mass measurements.

## 4. The general framework

As anticipated, within the MSSM there are in principle several new (non-Yukawa) sources of flavour-symmetry breaking. Some of them are highly constrained by precise data on various rare processes. However, we are still far from being able to conclude that the MFV hypothesis, namely the absence of new sources of flavour-symmetry breaking, is the only viable option.

Focusing the attention on the quark sector, we can identify five independent sources of flavour-symmetry breaking in the matrices $\mathbf{M}_Q^2$, $\mathbf{M}_U^2$, $\mathbf{M}_D^2$, $\mathbf{A}_U$, and $\mathbf{A}_D$. Their non-trivial parts (terms not proportional to the identity matrix) introduce breaking terms of the

$$G_{\text{flavour}}^{\text{quark}} = \mathrm{SU}(3)_{Q_L} \times \mathrm{SU}(3)_{U_R} \times \mathrm{SU}(3)_{D_R} \tag{4.1}$$

subgroup of $\mathrm{SU}(3)^5$ transforming respectively as

$$\mathbf{M}_Q^2 \sim (8,1,1)\,, \quad \mathbf{M}_D^2 \sim (1,1,8)\,, \quad \mathbf{A}_D \sim (\bar{3},1,3)\,, \tag{4.2}$$
$$\mathbf{A}_U \sim (\bar{3},3,1)\,, \quad \mathbf{M}_U^2 \sim (1,8,1)\ . \tag{4.3}$$

In the literature, there exist several phenomenological analyses of such terms, typically expressed as upper bounds on the mass-insertion couplings $(\delta^{U,D}_{\text{AB}})_{ij}$, defined as in eq. (3.7). In particular, $\epsilon_K$ and $\Delta M_K$ imply stringent bounds on all the 1–2 down-type mass insertions (limits in the $10^{-4}$–$10^{-3}$ range for squark masses below 500 GeV) [33]; bounds in the $10^{-2}$ range for all the 1–3 down-type couplings follow from $\Delta M_{B_d}$ and $A_{\text{CP}}(B_d \to J/\psi K)$ [34]; bounds in the $10^{-2}$–$10^{-1}$ range are derived on the 2–3 down-type couplings from $B \to X_s\gamma$ [35] (which strongly constrains LR terms) and recently also by $\Delta M_{B_s}$ [35] (which is also sensitive to LL and RR terms). All these stringent phenomenological limits have been derived analysing the impact of gluino-mediated amplitudes. As a consequence, the constraints concern only down-type mass matrices, or the three flavour-symmetry breaking structures in eq. (4.2). The bounds on the up-type soft-breaking terms, derived from

| | | | |
|---|---|---|---|
| Charginos: | $\mu = 500 \pm 10\,\mathrm{GeV}$ | $M_2 = 300 \pm 10\,\mathrm{GeV}$ | $\tan\beta = 2$–$4$ |
| Up-squarks: | $M_{\tilde{u}_R} = 600 \pm 20\,\mathrm{GeV}$ | $M_{\tilde{q}_L} = 800 \pm 20\,\mathrm{GeV}$ | $A_0 = 1$ TeV |
| Other mass terms: | $M_1 = 500\,\mathrm{GeV}$ | $M_{\tilde{d}_R} = M_{\tilde{l}} = M_3 = M_{H^+} = 2$ TeV | |

**Table 2:** Basic choice of the flavour-conserving parameters used in figures 4–8.

chargino amplitudes, are substantially weaker (see e.g. ref. [36] for a recent analysis in the 2–3 sector).

Interestingly, within the SM the only large breaking of the flavour symmetry appears in the up-sector, or in the up-type Yukawa coupling transforming as $(\bar{3}, 3, 1)$. It is therefore quite natural to conceive supersymmetric scenarios where the three flavour-symmetry breaking structures in eq. (4.2) are very small (in agreement with observations) and sizable non-minimal breaking terms appear only in the up-type structures in eq. (4.3), especially in the $(\bar{3}, 3, 1)$ sector. As we shall show in the rest of this section, this scenario is perfectly compatible with all existing constraints on $B$ and $K$ physics, and rare $K$ decays are the most useful tools to probe it in the future.

### 4.1 Definition of the model

We consider a non-minimal scenario where $\mathbf{M}^2_Q$, $\mathbf{M}^2_D$, and $\mathbf{M}^2_U$ have an approximate MFV structure, while $\mathbf{A}_U$ contains sizable non-minimal flavour-breaking terms of $\mathcal{O}(\lambda)$, where $\lambda \approx 0.22$ is the Cabibbo angle. More precisely, we assume that in the super-CKM basis $\hat{\mathbf{M}}^2_{\tilde{u}LL}$ and $\hat{\mathbf{M}}^2_{\tilde{u}RR}$ have the form in eq. (3.9), while

$$\hat{\mathbf{M}}^2_{\tilde{u}LR} = [\mathbf{A} - \cot\beta\mu^*]\,\mathbf{m}_u\,, \tag{4.4}$$

where

$$(\mathbf{A})_{33} \equiv A_0\,, \qquad |(\mathbf{A})_{i\neq j}| \leq \lambda A_0\ . \tag{4.5}$$

This non-minimal structure is naturally consistent with vacuum stability bounds [26]. We note that structure of the type (4.4) also for $\hat{\mathbf{M}}^2_{\tilde{d}LR}$ (or $\mathbf{A}_D$), with $\mathbf{m}_u \leftrightarrow \mathbf{m}_d$, is generally consistent with the existing bounds from gluino-mediated FCNC amplitudes (given the smallness of $\mathbf{m}_d$). However, in order to isolate the effects induced by $\mathbf{A}_U$, in the following we concentrate on the case where $\mathbf{A}_D$ is aligned to the corresponding Yukawa matrix.

In order to compare the sensitivity of various FCNC observables to the new sources of flavour-symmetry breaking, we fix the flavour-conserving parameters of the model to some reference values, and study the dependence of the observables on the $A_{ij}$ terms by performing simple random scans.

This way we simulate somehow a post-LHC scenario, where most of the flavour-conserving parameters of the model are known, because of the progress at the high-energy frontier, while precision measurements of rare decays can be used to determine the flavour structure of the model.

### 4.2 Numerical analysis

The basic choice of the flavour-conserving parameters is shown in table 2. For simplicity, a high scale (2 TeV) has been chosen for all the parameters which play a minor role in chargino up-squark amplitudes. The results depend very little from this choice, provided the minimum value of down-squark/slepton masses is above 500 GeV. We have also assigned a small error to the key mass parameters of the chargino up-squark/sector, in order to investigate the sensitivity of flavour-changing observables to the precision on mass terms. Given the smallness of the Yukawa couplings of the first 2 generations, the only relevant $A_{ij}$ terms are $A_{13}$, $A_{23}$, and $A_{33} \equiv A_0$. As in the previous section, we have varied the free parameters of the model checking the consistency with direct experimental constraints on squark and gaugino masses, the existing constraints from rare processes, and vacuum stability. In particular, we have required relative non-standard contributions to $\mathcal{B}(B \to X_s\gamma)$ not to exceed 15%. The relative non-standard contributions to $\epsilon_K$ and $\Delta M_d$ have been constrained to be within 10%, except when these observables are explicitly shown in the plots to illustrate the correlations with $K \to \pi\nu\bar{\nu}$. Contrary to the previous section, here only simple random scans of the free parameters have been performed since maximal effects are not searched for.

The numerical results obtained for the two $K \to \pi\nu\bar{\nu}$ modes, in comparison with $B_d \to X_s\ell^+\ell^-$, $B_d \to \mu^+\mu^-$, $\Delta M_{B_d}$, and the CP-violating observables $\epsilon_K$ and $A_{\rm CP}(B_d \to J/\psi K)$, are shown in figures 4–8. The main features resulting from this numerical study can be summarized as follows:

- The non-standard effects induced by these chargino-mediated amplitudes, in the presence of non-MFV up-type $A$ terms, are maximal in the two $K \to \pi\nu\bar{\nu}$ decays. The dominance of $K \to \pi\nu\bar{\nu}$ holds in comparison with other $K$- and $B$-physics FCNC amplitudes, both in CP-conserving (figure 4) and in CP-violating observables (figure 6).

  Note that the non-standard effect in $\mathcal{B}(K \to \pi\nu\bar{\nu})$ is not necessarily an enhancement with respect to the SM. For instance, for $A_{13} \approx 0$ one has $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \lesssim \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM}$, while in the same region of parameter space $\Delta M_{B_d}$ receives a $\approx 4\%$ positive correction (see figure 4). However, the important feature emerging from our analysis is that $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ has a much stronger dependence on $A_{13}$ with respect to $\Delta M_{B_d}$ (and the other $B$-physics observables).

- As can be noted from figures 4–6, despite the fact that the flavour conserving parameters of the model are completely fixed, the predictions for the FCNC observables are still affected by a large uncertainty. This is due to the free parameters hidden in phases and moduli of $A_{13}$ and $A_{23}$ (see figure 5). For this reason, a complete determination of the model requires several precision measurements in the FCNC sector. Ideally, the two $K \to \pi\nu\bar{\nu}$ rates and two clean FCNC observables in the $B$ system (such as the rates of the two $B_{s,d} \to \mu^+\mu^-$ modes).

- As can be seen in figure 5, even setting $A_{23} = 0$, the dependence of $\mathcal{B}(K \to \pi\nu\bar{\nu})$ on $A_{13}$ is quadratic, contrary to the approximate linear dependence of the other FCNC observables. This quadratic dependence, which is the main reason of the dominance of non-MFV chargino-mediated amplitudes in $K \to \pi\nu\bar{\nu}$, can be explained in terms of the double-mass insertion mechanism of ref. [11].

To better understand this effect, we denote by $A_{ij}$ ($\bar{A}_{ij}$) the $A$ terms in the super-CKM basis (the squark basis of refs. [10, 11]). It is easy to realize that if $A_{23} = 0$ and $A_{13} \neq 0$ in the super-CKM basis, then both $\bar{A}_{13}$ and $\bar{A}_{23}$ are not vanishing because of the non-diagonal entries of the CKM matrix. In particular,

$$\bar{A}_{23} = V_{2j} A_{j3} \approx V_{21} A_{13} \ . \tag{4.6}$$

A similar mechanism occurs if $A_{13} = 0$ and $A_{23} \neq 0$. Thus, barring fine-tuned scenarios, there is always a sizable effective double mass-insertion coupling, given by

$$\left(\bar{\delta}^U_{\rm RL}\right)^*_{32} \left(\bar{\delta}^U_{\rm RL}\right)_{31} \propto \bar{A}_{23}\bar{A}^*_{13} \approx V_{12}|A_{23}|^2 + V_{21}|A_{13}|^2 + A_{23}A^*_{13} \tag{4.7}$$

Since the double mass-insertion enhancement is effective only in the kaon system (because of its double-CKM suppression [11]), rare $K$ decays are naturally the best probe of any non-MFV structure in $\mathbf{A}_U$.

A further check of the dominance of the double mass insertion mechanism is shown in the lower plot of figure 6, for the CP-violating channel $K_{\rm L} \to \pi^0\nu\bar{\nu}$. Here both $A_{23}$ and $A_{13}$ are non vanishing, and regions corresponding to specific choices of their CP-violating phases are outlined with different colors (gray scales). As can be noted, to a good approximation the non-standard effect depends only on their relative phase and –consistently with eq. (4.7)– is maximal when $A_{23}A^*_{13}$ is purely imaginary.

- At first sight, it is quite surprising that the non-MFV left-right mixing terms in eq. (4.5) are not excluded by the precise data on $\mathcal{B}(B \to X_s\gamma)$ and have a marginal impact also on $\mathcal{B}(B \to X_s\ell^+\ell^-)$ (see figure 4). However, this fact can easily be understood by noting that a non-vanishing $b \to s\gamma$ amplitude (generated by effective operators of the type $\bar{b}_R\sigma_{\mu\nu}s_L F^{\mu\nu}$ or $\bar{b}_L\sigma_{\mu\nu}s_R F^{\mu\nu}$) requires:

  i. odd number of chirality flips in the down sector;

  ii. odd total number of chirality flips summing up, down and flavour-independent terms.

This implies that the up-type left-right mixing terms in eq. (4.5) can contribute to the $b \to s\gamma$ transition only via amplitudes which have at least three chirality flips (the up-type trilinear term, one left-right mixing in the down sector, and a third $\mathrm{SU}(2)_L$ breaking term in order to recover the total helicity-violating structure). Since each chirality flip is associated with an insertion of the SM Higgs vev, this structure implies a strong suppression.

A phenomenological check of this statement is provided by the loose bounds on $(\delta^U_{\rm RL})_{32}$ extracted from $\mathcal{B}(B \to X_s\gamma)$ (see e.g. refs. [36, 37]).



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- An important property of the chargino-mediated non-MFV contributions is their slow decoupling in the limit of heavy superpartners in $K \to \pi\nu\bar{\nu}$ decays ($\sim m_{\rm SUSY}^{-2}$), compared to the fast decoupling in $\Delta F = 2$ observables ($\sim m_{\rm SUSY}^{-4}$). This property, which has been discussed in detail in ref. [12], is clearly illustrated in figure 7 in the case of CP-violating observables (a completely similar scenario holds in the CP-conserving case). This property provides another natural explanation of why we can still hope to observe sizable deviations from the SM in rare $K$ decays, despite the absence of non-standard effects in $\Delta M_{B_{d,s}}$, $A_{\rm CP}(B_d \to J/\psi K)$, and $\epsilon_K$.

- A well-defined prediction of this non-MFV scenario is a strict correlation between the non-standard contributions to $K_{\rm L} \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0 e^+e^-$, and $K_L \to \pi^0\mu^+\mu^-$. As shown in figure 8, the relative size of the effect is larger in the $K_{\rm L} \to \pi^0\nu\bar{\nu}$ case; however, observable deviations from the SM are expected also in the two $K_L \to \pi^0\ell^+\ell^-$ modes. This correlation is a consequence of the $Z\bar{s}d$ effective vertex common to the three decay modes, which encodes the dominant non-SM contributions in the limit of heavy sleptons. The detailed structure of the correlation is almost independent of the flavour-conserving parameters of the MSSM, for sufficiently heavy sleptons, but has a significant dependence of the CKM phase $\gamma$ (as shown in figure 8).

  If at least two of the rare $K_L$ channels could be measured with good precision, their correlation within this non-MFV scenario would provide a striking signature of the model.

## 5. Conclusions

The determination of the flavour structure of the MSSM — as well as of any TeV extension of the SM — is one of the components of the so-called *inverse problem* [38], which hopefully we will face soon with the start of the LHC program. While the direct exploration of the TeV scale will reveal some of the features of the new-physics scenario, the LHC program alone is unlikely to completely determine the structure of the new underlying theory. This statement is particularly true in the case of the flavour structure of the theory, whose model-independent determination requires new high-precision measurements also at low energies.

In this paper we have provided a quantitative illustration of this problem, analysing the impact of the two $K \to \pi\nu\bar{\nu}$ modes in determining the flavour structure of the MSSM. We have analysed the expectations for the branching ratios of these modes in two representative classes of the MSSM (as far as the flavour structure is concerned): the Minimal Flavour Violation scenario [16] and a scenario with $\mathbf{A}_U$ terms not aligned with the corresponding Yukawa coupling. In both these frameworks precise measurements of the two $\mathcal{B}(K \to \pi\nu\overline{\nu})$ turn out to be key ingredients to determine the structure of the model.

Within the MFV scenario, the deviations from the SM in the two $\mathcal{B}(K \to \pi\nu\overline{\nu})$ are naturally small (typically within 10%), but could saturate the model-independent bounds of ref. [31]. We have outlined some clean correlations between the possible non-standard effects in the two $\mathcal{B}(K \to \pi\nu\overline{\nu})$ and the values of stop and chargino masses which provide a distinctive signature of this scenario, or an efficient method to test the implementation of the MFV hypothesis in the MSSM.

The situation is certainly more interesting in the case of non-MFV up-type trilinear terms. Within this scenario, which is well motivated and perfectly compatible with all existing constraints from $B$ and $K$ physics, the two $\mathcal{B}(K \to \pi\nu\overline{\nu})$ could receive $\mathcal{O}(1)$ corrections with respect to the SM. We have indeed demonstrated that these rare $K$ decays, together with the two $K_L \to \pi^0\ell^+\ell^-$ modes, represent the most sensitive probe of any misalignment between $\mathbf{A}_U$ and the corresponding Yukawa coupling. The precise measurement of these rare decays is therefore a necessary ingredient for a model-independent reconstruction of the flavour structure of the MSSM soft-breaking terms.

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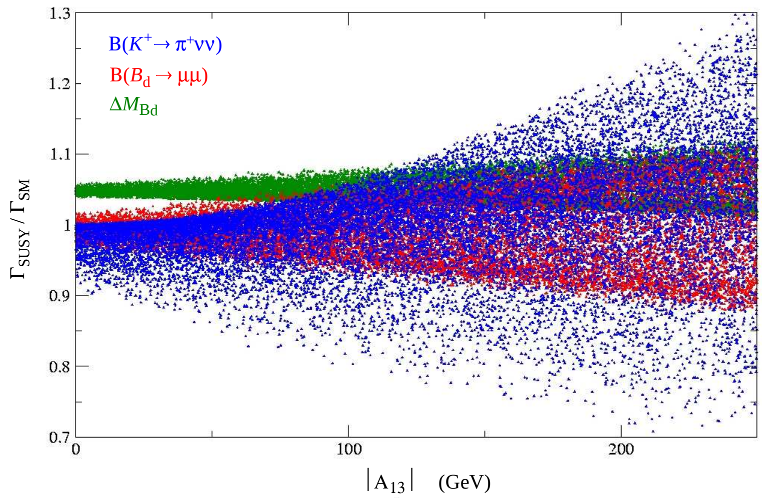


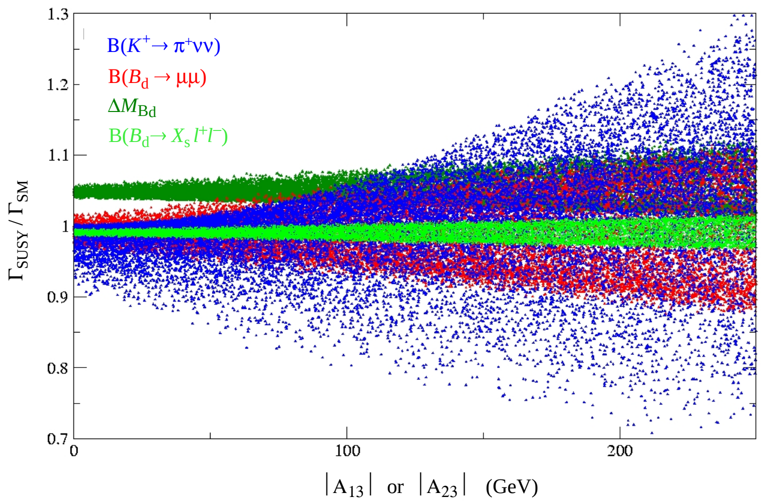


**Figure 4:** Dependence of various FCNC observables (normalized to their SM value) on the up-type trilinear terms $A_{13}$ and $A_{23}$, varied according to eq. (4.5). The flavour-conserving parameters of the model are fixed as specified in table 2. Upper plot: $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ (blue/dark gray), $\mathcal{B}(B_d \to \mu^+\mu^-)$ (red/gray lower-region), $\Delta M_{B_d}$ (green/gray upper-region) as a function of $A_{13}$. Lower plot: same observables as in the upper plot, with the superposition of $\mathcal{B}(B_d \to X_s\ell^+\ell^-)$ (light green/light gray) plotted as a function of $A_{23}$ (instead of $A_{13}$).

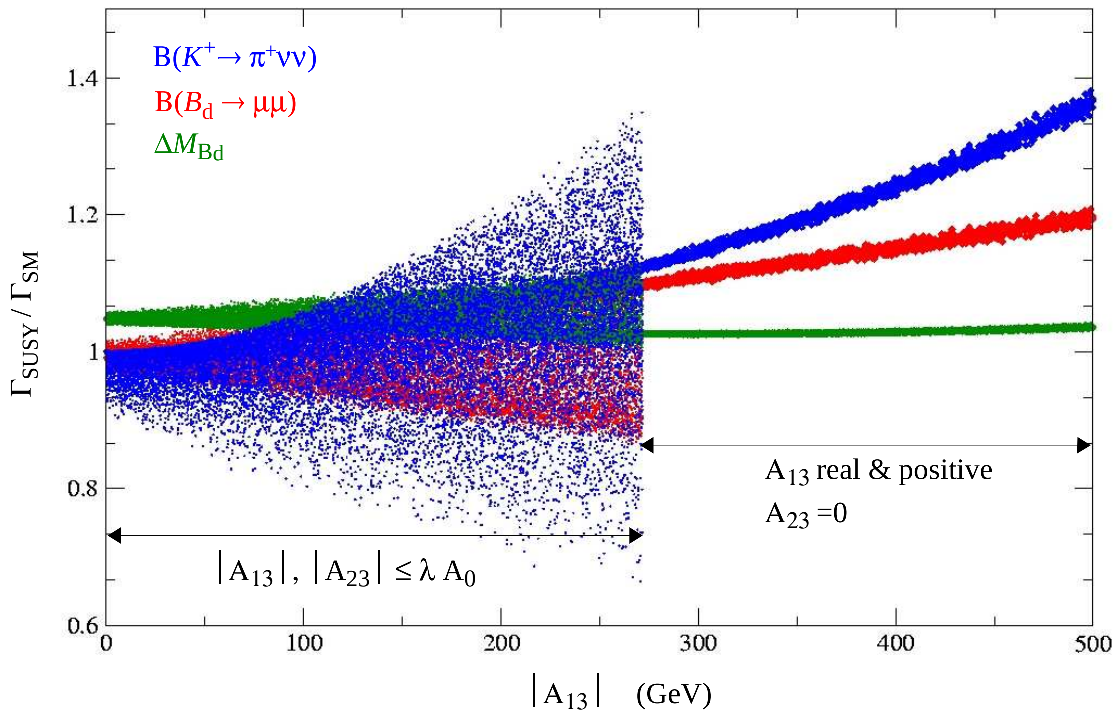


**Figure 5:** Anatomy of the SUSY contributions to $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$, $\mathcal{B}(B_d \to \mu^+\mu^-)$ and $\Delta M_{B_d}$ changing the range of variability of $A_{13}$ and $A_{23}$. Notations and other conventions as in figure 4.

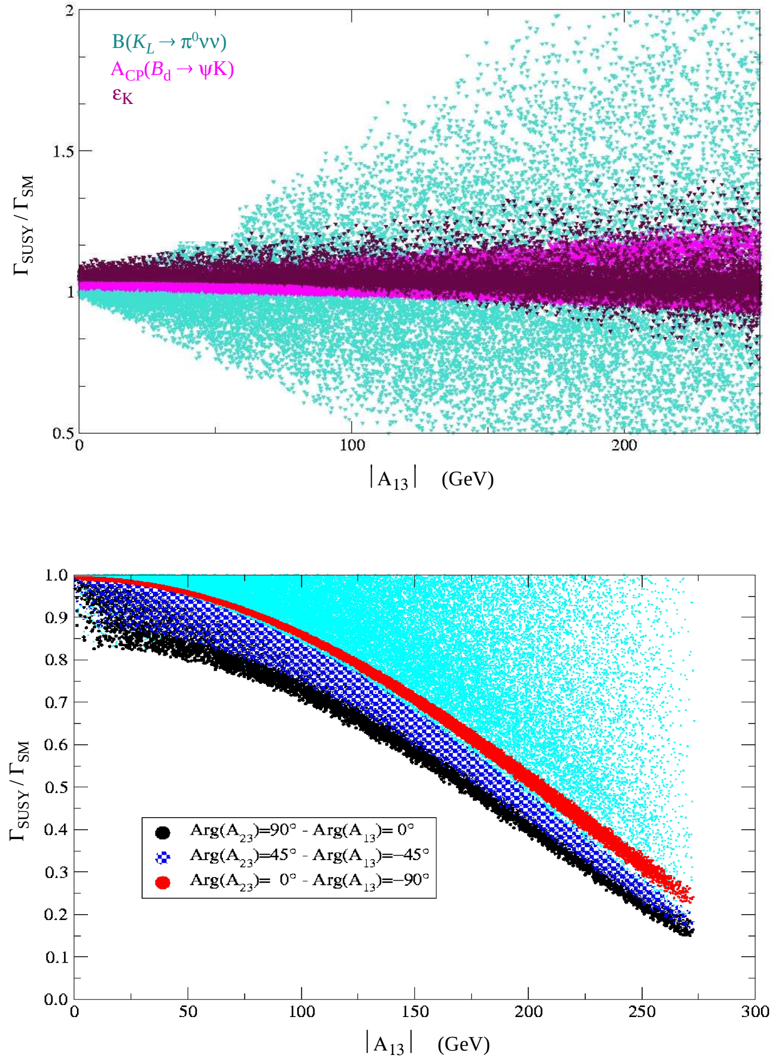


**Figure 6:** Impact of chargino-mediated amplitudes on CP-violating observables. Upper plot: comparison of $\epsilon_K$ (bordeaux/dark gray), $A_{\rm CP}(B_d \to J/\psi K)$ (violet/gray) and $\mathcal{B}(K_{\rm L} \to \pi^0 \nu \bar{\nu})$ (light blue/light gray). Lower plot: dependence of the non-standard contributions to $\mathcal{B}(K_{\rm L} \to \pi^0 \nu \bar{\nu})$ on the CP-violating phases of $A_{13}$ and $A_{23}$ in the region of destructive interference between SUSY and SM contributions (light-blue/light-gray dots correspond to no constraints on the phases). The trilinear terms are varied according to eq. (4.5); all the other supersymmetric parameters are fixed as in table 2.

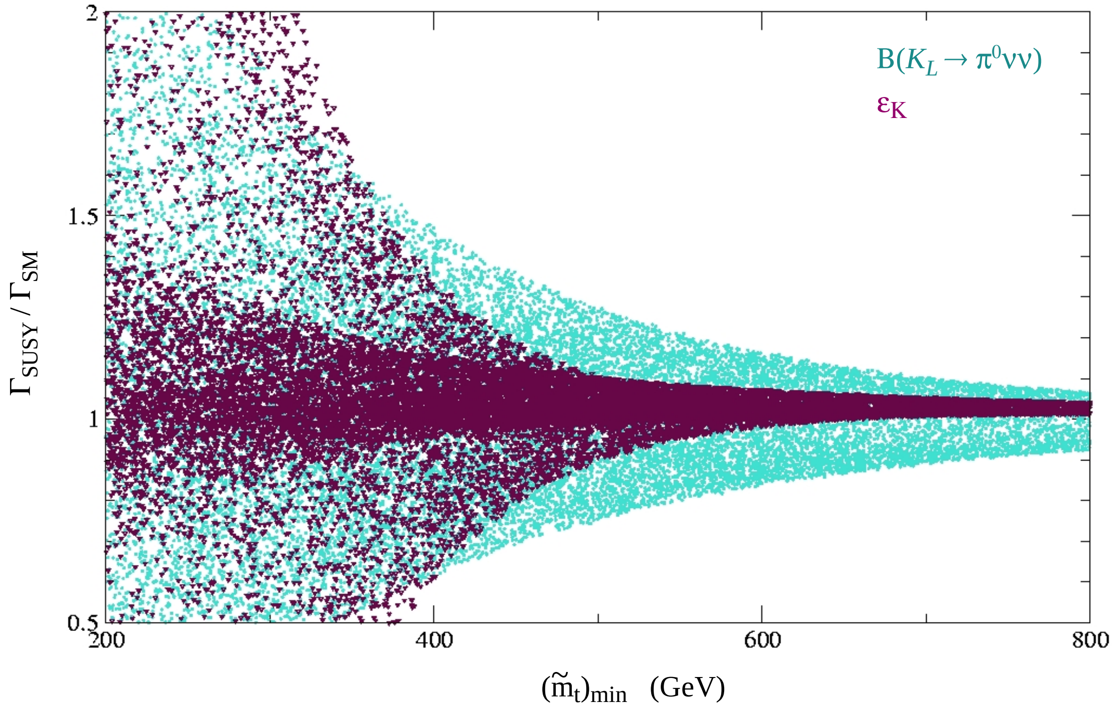


**Figure 7:** Comparison of the decoupling of non-standard contributions to $\epsilon_K$ (bordeaux/dark gray) and $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ (light blue/light gray) in the limit of heavy supersymmetric particles. The scatter plots are obtained varying $A_{13}$ and $A_{23}$ according to eq. (4.5), $M_{\tilde{u}_R}$ in the interval 200–1000 GeV, and fixing all the other mass parameters as in table 2. The horizontal axis denotes the lightest up-type squark mass.



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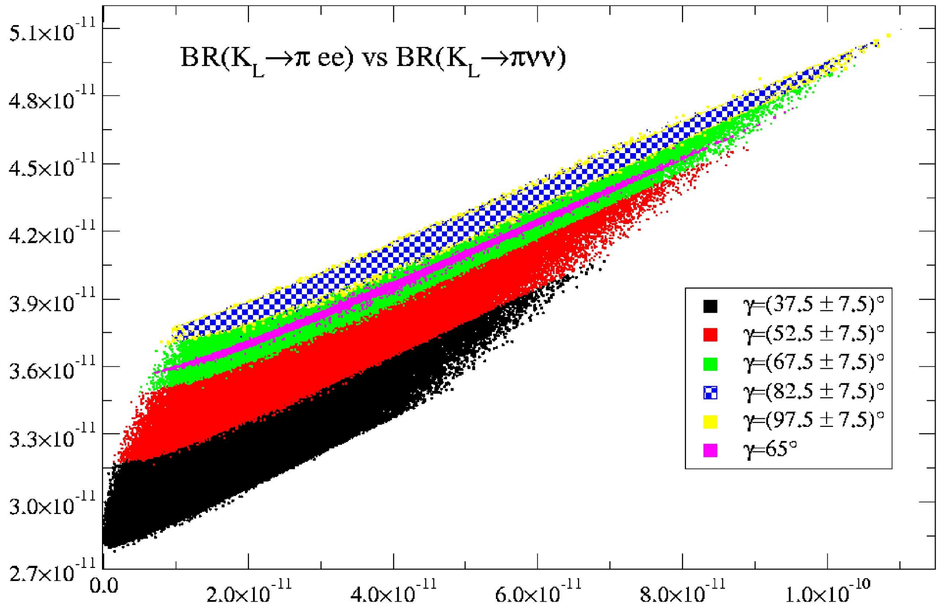


**Figure 8:** Correlation between $\mathcal{B}(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ (horizontal axis) and $\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ (vertical axis) in the MSSM scenario with non-minimal $A_{13}$ and $A_{23}$, varied according to eq. (4.5) (other parameters are fixed as in table 2). Different colors (gray scales) correspond to different values of the CKM phase $\gamma$.